A Fuzzy Hopfield Neural Network for Medical Image Segmentation

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***Abstract*—In this paper, an unsupervised parallel segmentation approach using a fuzzy Hopfield neural network (FHNN) is proposed. The main purpose is to embed fuzzy clustering into neural networks so that on-line learning and parallel implementation for medical image segmentation are feasible. The idea is to cast a clustering problem as a minimization problem where the criteria for the optimum segmentation is chosen as the minimization of the Euclidean distance between samples to class centers. In order to generate feasible results, a fuzzy c-means clustering strategy is included in the Hopfield neural network to eliminate the need of finding weighting factors in the energy function, which is formulated and based on a basic concept commonly used in pattern classification, called the "within-class scatter matrix" principle. The suggested fuzzy c-means clustering strategy has also been proven to be convergent and to allow the network to learn more effectively than the conventional Hopfield neural network. The fuzzy Hopfield neural network based on the within-class scatter matrix shows the promising results in comparison with the hard c-means method.**

I. Introduction

THE IMAGE segmentation that divides the image into meaningful subregions is an indispensable step in image analysis. In medical image analysis, the process of identifying the tissue organization from the human body in X-ray computed tomography (CT), magnetic resonance (MR), positron emission tomography (PET), and mammographic imaging is a very important process. A number of algorithms based on approaches such as histogram analysis, regional growth, edge detection, and pixel classification have been proposed in other articles [1]–[6]. In general, a thresholding algorithm is one that determines a threshold value based on a certain criteria. In the global thresholding method, the entire image is segmented with a single threshold value. A local thresholding method is one that thresholds a given image into subregions and determines a threshold value for every subregion. A regional growth is a technique that starts at the known pixel points and extends to all neighboring pixels that are similar in gray level, color, texture, or other properties in order to form a complete region. In the edge detection technique, local discontinuities are detected first and then connected to form complete boundaries. A pixel classification approach is one that classifies pixels into associated regions based on their gray levels. These methods generally utilize local information (i.e., the gray-level values of the neighboring pixels) and/or global information (i.e., the overall gray-level distribution of the image) for image segmentation. As examples, a segmentation of brain CT images based on regional growth and the nearest neighbor principle is proposed by Sandor *et al.* [7] and a segmentation algorithm to detect intensity basins in gray-scale images is described by Ngan *et al.* [8].

In addition, algorithms using the neural network technique have also been investigated in edge detection and image segmentation problems. Recently, an edge detection algorithm based on the Hopfield neural network was proposed by Chao and Dhawan [9]. Tan *et al.* [10] also presented an edge detection approach based on a cost minimization problem using simulated annealing. And a competitive learning network was applied to the color image segmentation [11], in which a competitive learning algorithm is presented for clustering the color space based on the least squares error criteria. A three-dimensional (3-D) constraint satisfaction neural network for medical image segmentation is presented by Chen *et al.* [12]. The segmentation of MR images using the probabilistic neural network is discussed by Morrison *et al.* [13], and the segmentation of MR brain images using mean field simulated annealing is elaborated by Snyder *et al.* [14]. Dhawan and Arata presented a self-organizing feature map algorithm for the segmentation of a medical image with competitive learning [15]. Their network was composed of two-dimensional (2-D) input (gray-level image) and output (segmented feature map) layers for combining a local contrast as well as the global gray-level distribution information to find out the desired regions.

Image segmentation can be considered as a clustering process in which the pixels are classified to the specific regions based on their gray-level values and spatial connectivity. In addition to the neural network-based technique, the fuzzy set has also been demonstrated to address segmentation problems in this paper. In this paper, a new segmentation method using a fuzzy Hopfield neural network (FHNN) based upon the pixel classification for medical image segmentation is proposed. This approach is different from the previous ones, in that a fuzzy reasoning strategy is added into a neural network. In FHNN, the problem of the image segmentation is regarded as a process of the minimization of a cost function. This cost function is defined as the Euclidean distance between the gray levels in a histogram to the cluster centers represented in the gray levels. The structure of this network is constructed as a 2-D fully interconnected array with the columns representing the

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number of classes and the rows representing the gray level of pixels taken as training samples. However, a training sample does not necessarily belong to one class. Instead, a certain degree of class membership is associated with every sample. In FHNN, an original Hopfield network is modified and the fuzzy c-means clustering strategy is added. Consequently, the energy function may be quickly converged into a local minimum, in order to produce a satisfactory resulting image. Compared with conventional techniques, the major strength of the presented FHNN is that it is computationally more efficient due to the inherent parallel structures. In a simulation study, the FHNN is demonstrated to have the capability for medical image segmentation and shown the promising results in comparison with the hard c-means method.

The remainder of this paper is organized as follows. Section II reviews the fuzzy clustering techniques; Section III proposes the medical image segmentation using a FHNN; Section IV shows the convergence of the FHNN on the mathematical derivations; Section V demonstrates the parameters on the performance of the FHNN; Section VI presents several experimental results; and finally, Section VII gives the discussion and conclusions.

II. Fuzzy Clustering Techniques

Clustering is a process for classifying the objects or patterns in such a way that the samples within a class are more similar to one another than samples belonging to other classes. Similarity measures employed to classify samples depend on the object characteristics, e.g., distance, vector, entropy, etc. Generally speaking, a clustering technique can be classified into either a one-shot partition or a hierarchical partition. A one-shot partition clustering method can be called a global partition process that generates a desired partition in one step to complete pattern classification, whereas a hierarchical partition clustering method can be referred to as a local partition process that decomposes a partition procedure into multiple stages so that the two most similar clusters are merged locally in one stage. Then, all locally merged clusters in different stages are hierarchically organized into a nested sequence of groups. There have been many applications based on clustering paradigms, some of these applications include image segmentation, speech recognition, and data compression. Many clustering strategies have also been used, such as the hard clustering algorithm and the soft (fuzzy) clustering algorithm, each of which has its own special characteristics. For example, the problem with the hard clustering algorithm, c-means [16], [17], is that it will cause the objective function to converge into a local minimum. Rather than assigning a sample to one and only one class, the fuzzy clustering method assigns the sample with a number, n, between zero and one described as a membership function. The work presented in this paper is an artificial neural network approach, based on the fuzzy c-means strategy, which can be viewed as a hierarchical partition method for medical image segmentation.

Since the introduction of the fuzzy set theory in 1965 by Zadeh, it has been applied in a variety of fields, such as medical image analysis [18]. The theory of fuzzy logic provides a mathematical framework to capture the uncertainties associated with human cognition processes.

Fuzzy clustering strategies have been studied by several authors [19]–[21]. According to Bezdek [22], the fuzzy clustering problem will be described as follows: let $Z = \{z_1, z_2, \cdots, z_x, \cdots, z_n\}$ be a finite unclassified data set, where z_x is an p-dimensional training sample. The fuzzy clusters $c_1, \cdots, c_c$ are generated by partition Z in accordance with the membership functions matrix $U = [\mu_{x,i}]$. The element $\mu_{x,i}$ denotes the degree of possibility that a z_x belongs to an ith fuzzy cluster. The fuzzy approach, which is similar to the usual clustering techniques, is to minimize the criteria in the least squared error sense. The objective function is defined as

$$J_{\mathrm{FCM}} = \frac{1}{2} \sum_{j=1}^{c} \sum_{x=1}^{n} (\mu_{x,j})^m |z_x - w_j|^2 \tag{1}$$

where $|.|$ is the Euclidean distance. The fuzzy c-means (FCM) clustering algorithm was first introduced by Dunn [23], and a related formulation and algorithm was proposed by Bezdek [24]. The FCM algorithm, being a well-known and powerful method in clustering analysis, is reviewed as follows.

A. Fuzzy c-Means Algorithm

Step 1) Initialize the number of classes c and $U(t)$, the fuzzification parameter

$$m\,(1 \le m < \infty), \text{and the value } \varepsilon > 0. \text{ Set } t = 0;$$

Step 2) Calculate the class center matrix $W^{(t)} = [w_1, w_2, \cdots, w_c]$ using $U^{(t)}$

$$w_i = \frac{1}{\sum_{x=1}^{n} (\mu_{x,i})^m} \sum_{x=1}^{n} (\mu_{x,i})^m z_x, \quad \text{for every } i; \tag{2}$$

Step 3) Calculate the membership matrix $U^{(t+1)} = [\mu_{i,j}]$using $W^{(t)}$

$$\mu_{x,i} = \left[\sum_{j=1}^{c} \left(\frac{|z_x - w_i|}{|z_x - w_j|} \right)^{2/(m-1)} \right]^{-1}, \quad \text{for every } x \text{ and } i; \tag{3}$$

Step 4) Compute $\Delta = \max[|U^{(t+1)} - U^{(t)}|]$. If $\Delta > \varepsilon$, then go to Step 2) and set $t = t+1$; otherwise stop the process.

The value m, prechosen as any value from one to ∞, is called the fuzzification parameter (or exponential weight), and it reduces the noise sensitivity in the computation of the class centers. Notice that the algorithm reduces to a crisp clustering algorithm in the case that $m = 1$. In addition, m reduces the influence of small $\mu_{i,j}$ compared to that of large $\mu_{i,j}$. The larger the value m, the stronger is this influence.

III. FUZZY HOPFIELD NEURAL NETWORK

Over the last few years, the Hopfield [25], [26] neural network has been studied extensively with its features of simple architecture and potential for parallel implementation. Polygonal approximation using a competitive Hopfield neural network was demonstrated by Chung *et al.* [27]. In [27], a 2-D discrete Hopfield neural network used the winner-takes-all learning to eliminate the need for finding weighting factors in the energy function. Endocardial boundary detection using the Hopfield neural network was described by Tsai *et al.* [28], Washizawa [29] applied the Hopfield neural network to emulate saccades, and optimal guidance using the Hopfield neural network was presented by Steck *et al.* [30]. The Hopfield neural network is a well-known technique used for solving optimization problems based on the Lyapunov energy function. A conventional 2-D parallel Hopfield network for the classification problem is first reviewed. The network consists of $n \times c$ neurons which are fully interconnected neurons. Let $V_{x,i}$ denote the binary state of neuron (x, i) and $W_{x,i;y,j}$ be the interconnection weight between the neuron (x, i) and the neuron (y, j). A neuron (x, i) receives each neuron (y, j) with $W_{x,i;y,j}$ and a bias $I_{x,i}$ from outside can then be expressed by

$$Net_{x,i} = \sum_{y=1}^{n} \sum_{j=1}^{c} W_{x,i;y,j} + I_{x,i} \tag{4}$$

and the Lyapunov energy function of the two-dimensional Hopfield network is given by

$$E = -\frac{1}{2} \sum_{x=1}^{n} \sum_{y=1}^{n} \sum_{i=1}^{c} \sum_{j=1}^{c} V_{x,i} W_{x,i;y,j} V_{y,j} - \sum_{x=1}^{n} \sum_{i=1}^{c} I_{x,i} V_{x,i}. \tag{5}$$

Each column of the Hopfield neural network represents a class and each row represents samples in a proper class. The network reaches a stable state when the Lyapunov energy function is minimized. For example, a neuron (x, i) in a firing state (i.e., $V_{x,i} = 1$) indicates that sample z_x belongs to class i. But, in the fuzzy Hopfield neural network, a neuron (x, i) in a fuzzy state indicates that sample z_x belongs to class i with a degree of uncertainty described by a membership function.

In [12], each pixel in an $L \times L$ image can be considered as an object being assigned to one of M labels. Then, the constraint satisfaction neural network, proposed by Chen *et al.*, consists of $L \times L \times M$ neurons that can be conceived as a 3-D array for the image-segmentation problem. The number of neurons is dependent on image size; the larger the image size, the more neurons that are required. In this paper, the global histogram, but not the spatial connectivity information of the medical images, is employed for the process of the image segmentation.

In a 2-D image, each pixel is assigned one of n gray levels. If the number of subregions c is defined in advance, then the FHNN consists of $n \times c$ neurons that can be conceived as a 2-D array. Consequently, the number of neurons is independent of the image size. In this section, we will show that the medical image segmentation problem can be mapped onto a Hopfield neural network so that the cost function serves as the energy function of the network. The idea is to form the energy function of the network in terms of the intra-class energy function. In the pattern recognition application, the degree of natural association is expected to be high among members belonging to the same class and low among members of different clusters. In other words, the intraset (within-class) distance should be small. The proposed technique first assigns samples to their associated classes in such a manner that the Euclidean distance between arbitrary samples to their class center is minimized. This is referred to as the intra-class assignment. In linear discriminate analysis [31], the concept of within-class scatter matrix is widely used for class separability. The iteratively updated synaptic weight between the neuronal interconnections will gradually force the network to converge into a stable state where its energy function is minimized.

If we suppose that the number of classes, $c \geq 2$, is prespecified, and let $Z = \{z_1, z_2, \cdots, z_m\}$ be a set of samples to be classified. $M_{n \times c}$ is the set of all real $n \times c$ matrices where a matrix $U = [\mu_{x,i}] \in M_{n \times c}$ is called a fuzzy-c partition if it satisfies the following conditions:

$$\mu_{x,i} \in [0, 1], \quad \text{for all } x \text{ and } i \tag{6}$$

$$\sum_{i=1}^{c} \mu_{x,i} = 1, \quad \text{for all } x \tag{7}$$

$$0 < \sum_{x=1}^{n} \mu_{x,i} < n, \quad \text{for all } i \tag{8}$$

$$\sum_{x=1}^{n} \sum_{i=1}^{c} \mu_{x,i} = n. \tag{9}$$

We will let the brightness of a given image be represented by n gray levels. The frequency of occurrence of each gray level z_x will be denoted by p_x. Using the within-class scatter matrix criteria, the optimization problem can be mapped into a 2-D fully interconnected Hopfield neural network with the fuzzy c-means strategy for medical image segmentation. The total weighed input for neuron (x, i) and Lyapunov energy, as defined in (4) and (5), can be modified as

$$Net_{x,i} = \left[z_x - \sum_{y=1}^{n} W_{x,i;y,i} (\mu_{y,i})^m \right]^2 + I_{x,i} \tag{10}$$

and

$$E = \frac{1}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} p_x (\mu_{x,i})^m \left[z_x - \sum_{y=1}^{n} W_{x,i;y,i} (\mu_{y,i})^m \right]^2 - \sum_{x=1}^{n} \sum_{i=1}^{c} I_{x,i} (\mu_{x,i})^m \tag{11}$$

is the total weighed input received from the neuron (y, i) in row i, $\mu_{x,i}$ is the output state at neuron (x, i), and m is the fuzzification parameter. Each column of this modified Hopfield network represents a class and each row represents a sample (gray level) in a proper class. The network reaches a stable state when the modified Lyapunov energy function is minimized. For example, a neuron (x, i) in a maximum membership state indicates that sample z_x belongs to class i.

In order to generate an adequate classification with the constraints given by (6) and (9), we define the objective function as follows:

$$E = \frac{A}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} p_x(\mu_{x,i})^m \cdot \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y (\mu_{y,i})^m \right]^2 + \frac{B}{2} \left[\left(\sum_{x=1}^{n} \sum_{i=1}^{c} \mu_{x,i} \right) - n \right]^2 \tag{12}$$

where E is the total intra-class scatter energy that accounts for the scattered energies distributed by all samples in the same class and both z_x z_y are the trained gray levels at rows x and y, respectively.

The first term in (12) is the within-class scatter energy, which is the Euclidean distance between samples to the cluster center over c clusters. The second term, stemming from (9), guarantees that the n samples in Z can only be distributed among these c classes. More specifically, the second term, which is the penalty term, imposes constraints on the objective function and the first term minimizes the intra-class Euclidean distance from a sample to the cluster center in any given cluster.

As mentioned in [27], the quality of classification result is very sensitive to the weighting factors. Searching for optimal values for these weighting factors is expected to be time-consuming and laborious. To alleviate this problem, a Hopfield neural network with a fuzzy c-means clustering strategy, called FHNN, is proposed so that the penalty terms can be handled more efficiently. All the neurons on the same row compete with each other to determine which neuron is the maximum membership value belonging to class i. In other words, the summation of the membership states in the same row equals one, and the total membership states in all n rows equal n shown as (9). It ensures that only n samples will be classified into these c classes. The modified Hopfield neural network, FHNN, enables the scatter energy function to converge rapidly into a minimum value. Then, the scatter energy of the FHNN can be further simplified as

$$E = \tfrac{1}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} p_x(\mu_{x,i})^m \cdot \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y (\mu_{y,i})^m \right]^2. \tag{13}$$

By using (13), which is modification of (12), the minimization of energy E is greatly simplified since it contains only one term, and hence the requirement of having to determine the weighting factors A and B vanishes. Comparing (13) with the modified Lyapunov function (11), the synaptic interconnection weights and the bias input can be obtained as

$$W_{x,i;y,i} = \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y \tag{14}$$

and

$$I_{x,i} = 0. \tag{15}$$

By introducing (14) and (15) into (10), the input to neuron (x, i) can be expressed as

$$Net_{x,i} = \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y (\mu_{y,i})^m \right]^2. \tag{16}$$

Consequently, the input-output function (i.e., membership function) for the xth row is given as

$$\mu_{x,i} = \left[\sum_{j=1}^{c} \left(\frac{Net_{x,i}}{Net_{x,j}} \right)^{2/m-1} \right]^{-1} \quad \text{for all } i. \tag{17}$$

Using (13), (16), and (17), the FHNN can classify c classes in a parallel manner described as follows.

A. FHNN Algorithm

Step 1) Input a set of training gray levels $Z = \{z_1, z_2, \cdots, z_n\}$ and their associated frequencies of occurrence $P = \{p_1, p_2, \cdots, p_n\}$, fuzzification parameter m $(1 \leq m < \infty)$, the number of classes c, and randomly initialize the states for all neurons $U = [\mu_{x,i}]$ (membership matrix);

Step 2) Compute the weighted matrix $W = [W_{x,i;y,j}]$ using (14);

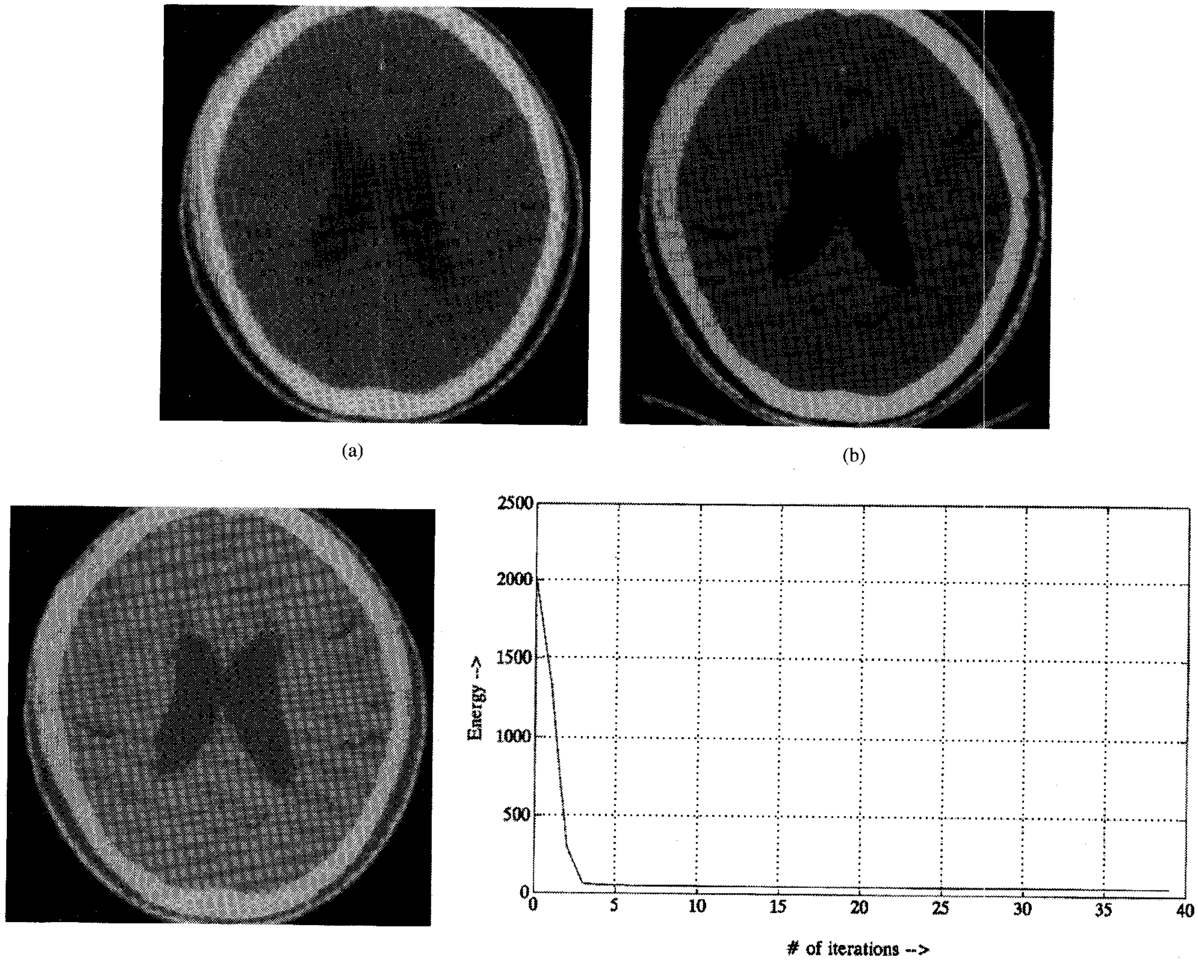
Step 3) Calculate the input to each neuron (x, i)

$$Net_{x,i} = \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y (\mu_{y,i})^m \right]^2;$$

Step 4) Apply (17) to update the neuron's membership value in a synchronized manner;

Step 5) Compute $\Delta = \max[|U^{(t+1)} - U^{(t)}|]$. If $\Delta > \varepsilon$, then go to Step 2), otherwise stop the process.

In Step 3), the inputs are calculated for all neurons. In Step 4), the fuzzy c-means clustering method is applied to determine the fuzzy state with the synchronous process. Here, a synchronous iteration is defined as an updated fuzzy state for all neurons using a software simulation.



(c) (d)

Fig. 1. (a) CT brain image for segmentation with $c = 5$. (b) Segmented result using the FHNN. (c) Segmented result using the c-means. (d) Energy curve in converging iterations in the FHNN.

IV. Convergence of the FHNN

Proof of the convergence of the FHNN is important because it guarantees that the network evolution will reach a stable state. The scatter energy function can be rewritten as follows:

$$E = \frac{1}{2}\sum_{i=1}^{c}\sum_{x=1}^{n} p_x(\mu_{x,i})^m \cdot \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y(\mu_{y,i})^m \right]^2. \quad (18)$$

due to $p_x(\mu_{x,i})^m \leq 1$; which implies that

$$E \leq \frac{1}{2}\sum_{i=1}^{c}\sum_{x=1}^{n} \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y(\mu_{y,i})^m \right]^2. \quad (19)$$

Equation (19) shows that the objective energy is less than or equal the total distance between training samples to the cluster centers. This proves that E is bounded from below.

Equation (18), the same as (1), is based on a least-squared errors criteria and is rewritten as follows:

$$E = \frac{1}{2}\sum_{i=1}^{c}\sum_{x=1}^{n} p_x(\mu_{x,i})^m |z_x - \omega_i|^2 \quad (20a)$$

and

$$\omega_i = \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n} p_h(\mu_{h,i})^m} p_y z_y(\mu_{y,i})^m \quad (20b)$$

where ω_i (center of cluster i) is the total interconnection weight received from all neurons y in the same column i. As proved in [32] and [33], ω_j is an iteratively chosen center in the same cluster and justified by the fact that

$$\sum_{x=1}^{n} (\mu_{x,i})^m |z_x - \omega_i|^2 = \sum_{x=1}^{n} (\mu_{x,i})^m |z_x - \omega_j|^2$$

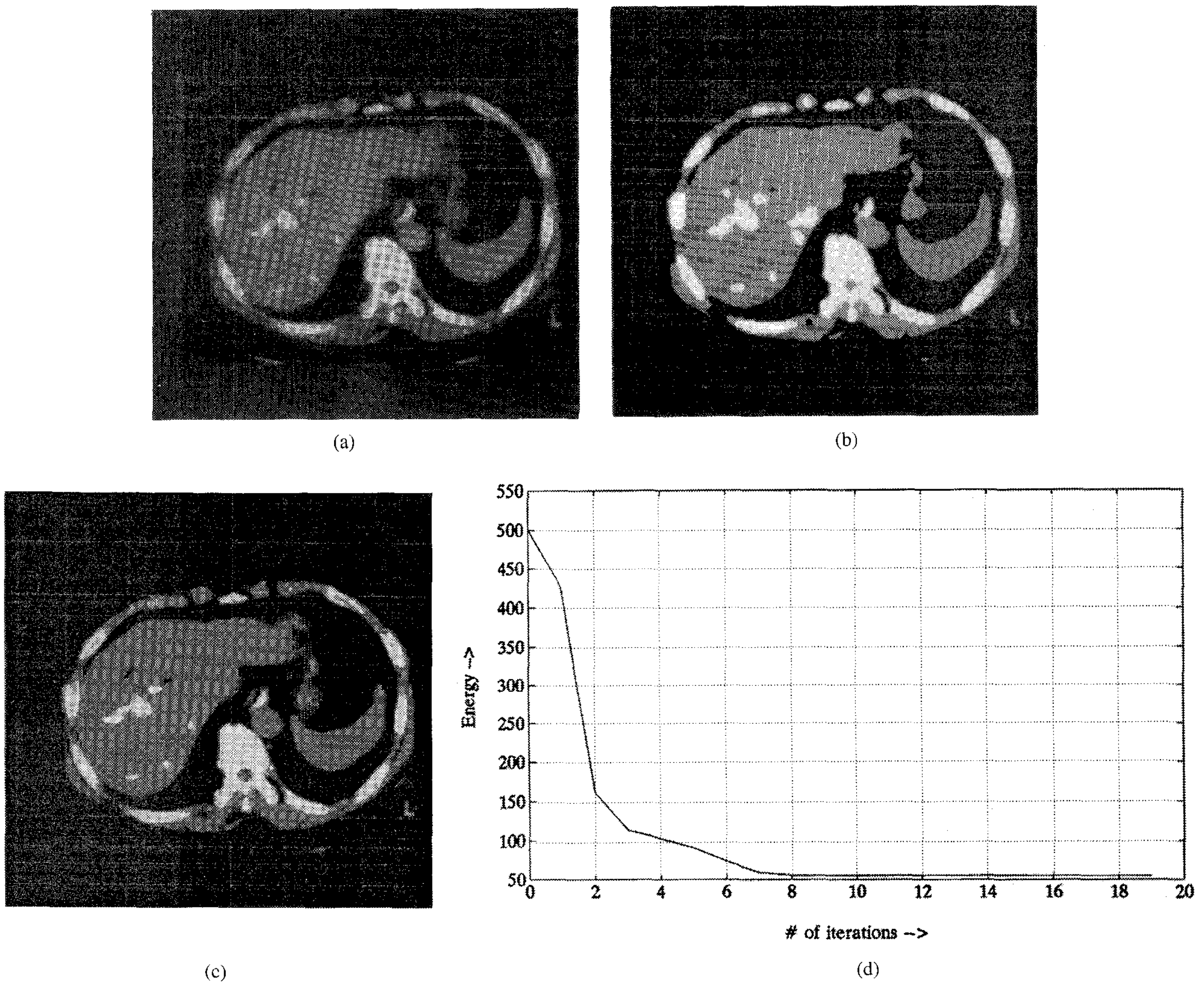


Fig. 2. (a) CT liver image for segmentation with $c = 3$. (b) Segmented result using the FHNN. (c) Segmented result using the c-means. (d) Energy curve in converging iterations in the FHNN.

$$+\left[\sum_{x=1}^{n}(\mu_{x,i})^m\right]|\omega_j-\omega_i|^2 \qquad (21)$$

so that

$$\sum_{x=1}^{n}(\mu_{x,i})^m|z_x-\omega_i|^2 \geq \sum_{x=1}^{n}(\mu_{x,i})^m|z_x-\omega_j|^2. \qquad (22)$$

Thus, the reassignment of a membership degree belonging to cluster i in training sample z_x will result in a decrease of the objective energy function whenever z_x is located closer to a feasible cluster center. Consequently, the FHNN will converge to a satisfactory result after several iterations of updating the reassignment matrix.

V. Parameters on the Performance of the FHNN

It can be found that the energy E is a function of fuzzification parameter m. There is no theoretical basis for an optimal choice of m. Bezdek [34] presented that the minimum of the objective function (20a) is strictly monotonical decreasing with m. To discuss the performance of the FHNN in which to change the weights, the negative of the gradient of E with respect to the weights ω_i is calculated as follows:

$$-\frac{\partial E}{\partial \omega_i} = \sum_{x=1}^{n} p_x(\mu_{x,i})^m(z_x-\omega_i) - \sum_{x=1}^{n} p_x\,\frac{m}{2}\,(\mu_{x,i})^{m-1}\,\frac{\partial \mu_{x,i}}{\partial \omega_i}\,|z_x-\omega_i|^2 \qquad (23)$$

where the derivative of $\mu_{x,i}$ with respect to ω_i can be obtained as

$$\frac{\partial \mu_{x,i}}{\partial \omega_i} = \frac{2\mu_{x,i}(1-\mu_{x,i})(z_x-\omega_i)}{(m-1)|z_x-\omega_i|^2}. \qquad (24)$$

Replacing the $\partial\mu_{x,i}/\partial\omega_i$ in (23) by (24), the gradient descent on the objective function with fuzzy units can be written as

$$-\frac{\partial E}{\partial \omega_i} = \sum_{x=1}^{n} p_x\gamma_m(z_x-\omega_i) \qquad (25)$$

where

$$\gamma_m = (\mu_{x,i})^m\left[1-\frac{m}{m-1}(1-\mu_{x,i})\right]. \qquad (26)$$

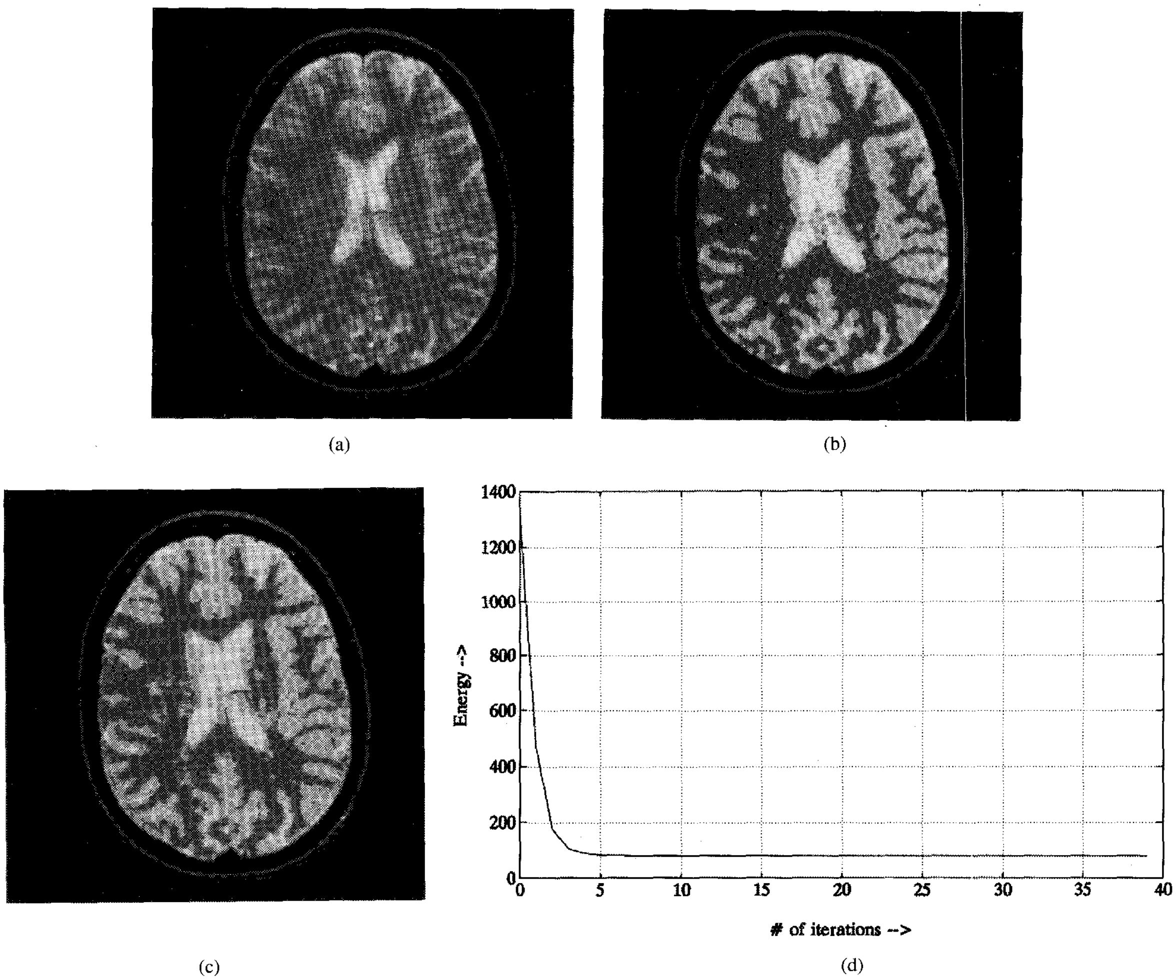


Fig. 3. (a) MR brain image for segmentation with $c = 4$. (b) Segmented result using the FHNN. (c) Segmented result using the c-means. (d) Energy curve in converging iterations in the FHNN.

For a proper fuzzification parameter m, the term γ_m, which is determined by the membership function $\mu_{x,i}$, controls the amount of the weight ω_i to be updated. Equation (26) is same as the definition in [35]. In [35], Jou indicated that $\gamma_m(\mu_{x,i})$ goes smoothly from zero to one as $\mu_{x,i}$ goes from zero to one at large value m ($m \geq 1.5$) and it makes a rather sudden rise from near $-\infty$ to one over a narrow range of $\mu_{x,i}$ near one at small value m. Membership functions for large value m are fuzzier than those for small value m, but the interconnection weights are updated slowly.

Threshold value ε, another important parameter, is used as a criterion to determine the performance of the objective function. The larger the threshold value ε, the less the number of iterations will, however, be the optimal membership function can not be found.

VI. Experimental Results

To see the capability of the proposed FHNN algorithm, five medical images with 256×256 pixels and 8-bit gray levels are tested with the hard c-means technique using MATLAB and C language on a Sun Sparc workstation. The first two medical images, shown in Figs. 1(a) and 2(a), are CT brain and liver images, respectively. The others, shown in Figs. 3(a), 4(a), and 5(a), are MR brain and chest images. For all experiments, the network-associated parameters such as fuzzification parameter (m) and threshold value (ε) are set to be 1.2 and 0.001 in the FHNN. Since the number of gray levels is 256 and the number of possible labels for each gray level in the FHNN is set to be c, the dimension of the neuron array is 256 in a row and c in a column. In all tested images, (a) shows the original image, (b) is the segmented one using the FHNN, (c) is the segmented result with the hard c-means, and (d) the energy curve in converging iterations in the FHNN. The curve in all examples clearly shows that the objective functions converge to a stable state rapidly after several synchronous iterations.

It is difficult to compare different image segmentation methods [6], [12]. Nevertheless, the major criteria for performance evaluation is whether the method can indicate interesting or important regions in the image. A segmentation method can therefore be declared successful if it can identify the desired and most important components. For instance, in Fig. 3, the segmented result can outline the CSF, the white matter, and the

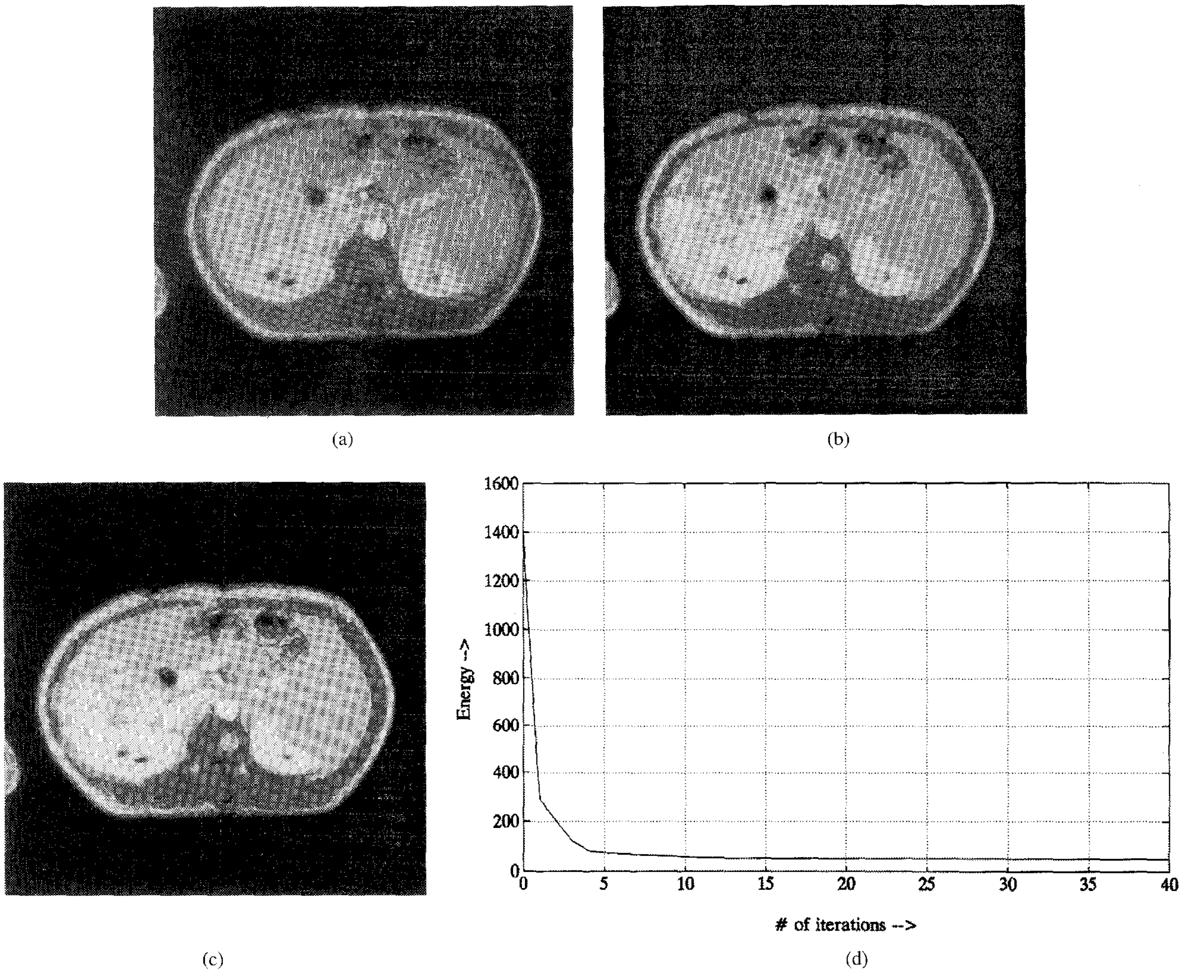


Fig. 4. (a) MR brain image for segmentation with $c = 4$. (b) Segmented result using the FHNN. (c) Segmented result using the c-means. (d) Energy curve in converging iterations in the FHNN.

gray matter from the transaxial MRI of the brain. In order to display the performance of the proposed FHNN, the uniformity measure proposed by Levine and Nazif [36] is used to compare the segmentation performance for the proposed FHNN and the hard c-means methods in this paper. For a given segmented image, the uniformity measure U_α is given by

$$U_\alpha = 1 - \left(\frac{\sum_{R_i \in \alpha} w_i \sigma_i^2}{M} \right) \tag{27}$$

where R_i is segmented region i and w_i is the weight associated with the contribution of region R_i to the measure. A_i is the total number of pixels in region R_i for $i = 1, 2, \cdots, c$. The variance for the gray levels in R_i is defined as

$$\sigma_i^2 = \sum_{(x, y) \in R_i} \frac{[f(x, y) - \overline{f}_i]^2}{A_i} \tag{28}$$

where $f(x, y)$ represents the gray level of the pixel (x, y) and $\overline{f}_i$ is the average gray level in region R_i. The value M is computed as

$$M = \left(\sum_{R_i \in \alpha} w_i \right) \cdot \frac{(f_{\max} - f_{\min})^2}{2} \tag{29}$$

with $f_{\max}$ = maximum gray level in region R_i and $f_{\min}$ = minimum gray level in region R_i, respectively. The uniformity measure for all segmented images using hard c-means and the proposed FHNN algorithms are computed and listed in Table I for comparison. The proposed FHNN algorithm produces a segmented images more promising than does the hard c-means.

Chung [27] indicated that the quality of the final solution is very sensitive to the values of weighting factors A and B, and searching for the optimal values would be time-consuming and laborious. The problem of determining the optimal values of the weighting factors is avoided in the FHNN. It is implied that this new approach is more efficient and versatile than the conventional Hopfield neural network for medical image segmentation. It is also noted that the resulting images are processed without human intervention. Thus, the experimental results can be regarded as near optimal and can be used for subsequent processes. Generally, the FHNN approach for

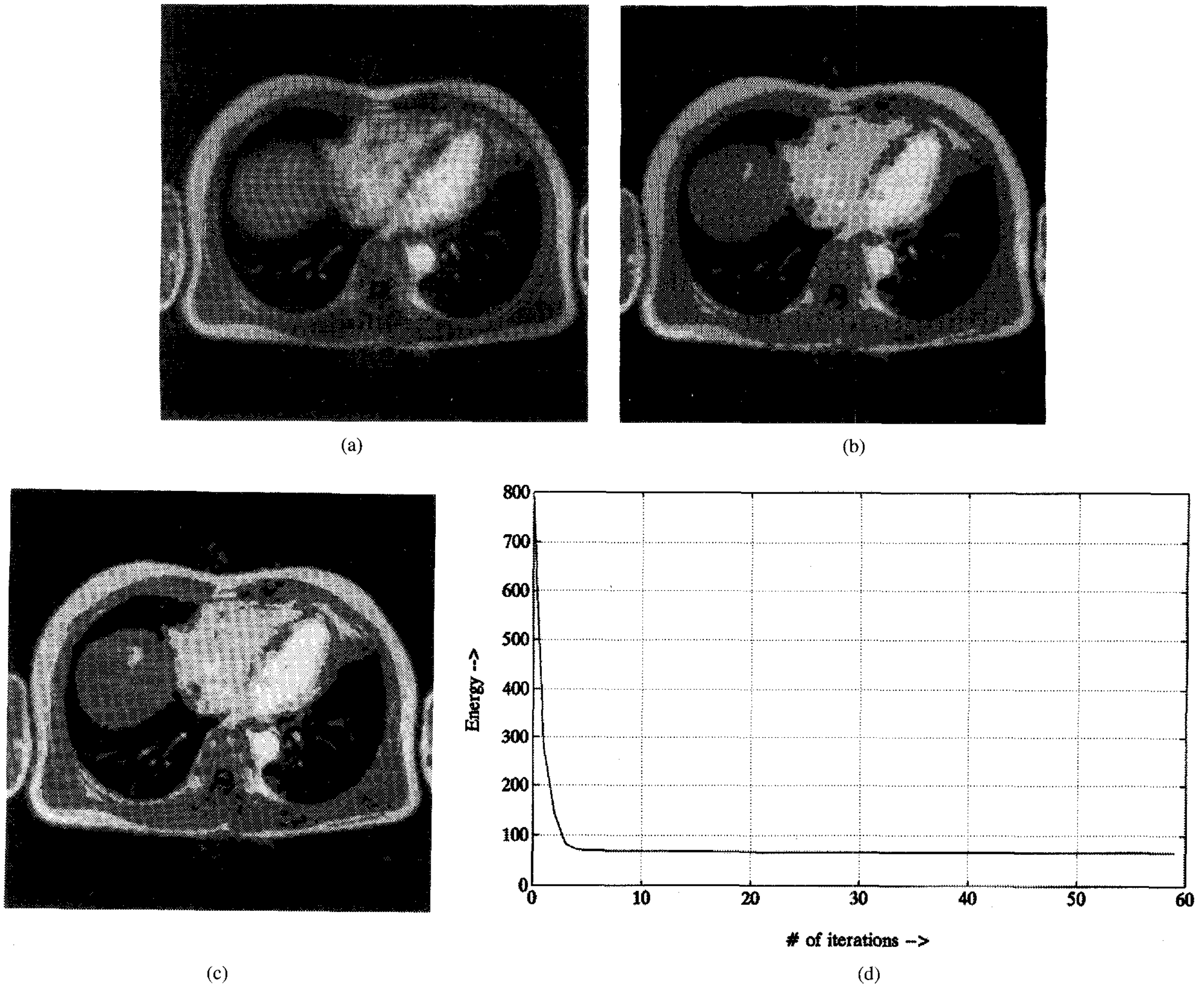


Fig. 5. (a) MR chest image for segmentation with $c = 4$. (b) Segmented result using the FHNN. (c) Segmented result using the c-means. (d) Energy curve in converging iterations in the FHNN.

TABLE I
UNIFORMITY MEASURE OF HARD C-MEANS AND THE PROPOSED ALGORITHMS IN TEST IMAGES

image	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5
c-means	0.994	0.863	0.913	0.805	0.991
FHNN	0.997	0.924	0.995	0.896	0.995

image segmentation needs much more computation time than conventional methods. However, due to the FHNN's highly interconnected and parallel abilities, computation time can be largely reduced by way of parallel processing.

VII. DISCUSSION AND CONCLUSION

A 2-D Hopfield neural network based on the within-class scatter matrix using the fuzzy c-means strategy for medical image segmentation has been presented in this paper. From the experimental results, the proposed FHNN algorithm produces a segmented images more promising than does the hard c-means. Moreover, the segmented results using the hard c-means seem to exhibit some artifacts. However, these artifact can be resolved using fuzzy reasoning strategy in the FHNN. In addition, the quantitative performance study was made difficult by the lack of a gold standard to compare with and the unavailability of ground truth for verification of the results. The performance evaluation has relied upon the radiologist. The network differs from the conventional Hopfield network in that a fuzzy c-means clustering strategy is imposed for updating the neuron states. In the conventional Hopfield network, a neuron (x, i) in a firing state indicates that sample z_x belongs to class i. But, in the FHNN, a neuron (x, i) in a fuzzy state indicates that sample z_x belongs to class i with a degree of uncertainty described by a membership function.

The energy function used for the FHNN is called the scatter energy function, which is formulated and based on a widely used concept in pattern classification. The fuzzy c-means method implemented by the FHNN greatly simplifies the scatter energy function so that there is no need to search for the weighting factors imposed on the original energy function. As a result, the proposed algorithm appears to converge rapidly to the desired solution. Since only the information of the global histogram, not the spatial connectivity, was used in the objective function, the number of nodes (256 by c for 256 gray levels) in the FHNN would be independent of the size of the image. When the spatial information is included, the size of the neuron array will be increased and the performance will be decreased over a multiple of 256 (if the size of image is 256×256). Moreover, the designed FHNN neural-network-based approach is a self-organized structure that is highly interconnected and can be implemented in a parallel manner. It can also easily be designed for hardware devices to achieve very high speed implementation.

To find the global minimum of objective function using the Hopfield neural network is still an interesting problem. The availability of dedicated neural network hardware incorporate simulated annealing with additional constraints and different initial states to rapidly generate optimal segmentation will be researched in the future.

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